# Integrated Mathematics I 3132

**Course description:**

This course is the first of three courses in a series that uses a more integrated approach to cover the same algebra and geometry concepts and skills that are included in the traditional three course series. The problem situations, models, and technology used will foster connections among the various strands of mathematics and develop concepts from multiple perspectives.

**Standard 1.0: Number and Operations**

Students will recognize, represent, model, and apply real numbers and operations verbally, physically, symbolically, and graphically.

**Learning Expectations:**

The student will:

1.1 demonstrate an understanding of the elements, subsets, properties, and operations of rational numbers;

1.2 demonstrate understanding of positive integer exponents and perform operations with expressions involving exponents;

1.3 connect physical, graphical, verbal, and symbolic representations of rational numbers;

1.4 connect physical, graphical, verbal, and symbolic representations of absolute value;

1.5 articulate, model, and apply the concept of inverse (i.e. opposites and reciprocals);

1.6 describe, model, and apply inverse operations;

1.7 perform operations on algebraic expressions and informally justify the procedures chosen;

1.8 apply matrix addition, subtraction, and scalar multiplication in real- world problems (e.g. inventory), using appropriate technology;

1.9 use a variety of notations appropriately (e.g., exponential, functional, square root);

1.10 select and apply an appropriate method (i.e. mental arithmetic, paper and pencil, or technology) for computing with real numbers, and use estimation to evaluate the reasonableness of the result.

**Student Performance Indicators:**

**At Level 1, the student is able to**

- select the best estimate for the coordinate of a given point on a number line (only rational);
- identify the opposite of a rational number;
- determine the square root of a perfect square less than 169;
- use exponents to simplify a monomial written in expanded form;
- apply order of operations when computing with integers using no more than two sets of grouping symbols and exponents 1 and 2;
- select a reasonable solution for a real-world division problem in which the remainder must be considered;
- compare and contrast the GCF and LCM of a set of numbers.

**At Level 2, the student is able to**

- probe the relationships among various subsets of the real number system;
- compare and contrast the GCF and LCM of a set of algebraic expressions;
- order a given set of rational numbers (both fraction and decimal notations);
- identify the reciprocal of a rational number;
- add and subtract algebraic expressions;
- multiply two polynomials with each factor having no more than two terms;
- use estimation to determine a reasonable solution for a tedious arithmetic computation;
- select ratios and proportions to represent real-world problems (e.g. scale drawings, sampling);
- perform operations on matrices using appropriate technology (addition, subtraction, and scalar multiplication).

**At Level 3, the student is able to**

- apply the concept of slope to represent rate of change in a real-world situation;
- scrutinize approximate values of real numbers such as pi and the square root of two.

**Sample Task:**

Students research the history of prime numbers and their uses.

**Linkages:**

Make connections to types of numbers used in science, social studies, and finance.

**Standard 2.0: Algebra**

Students will describe, extend, analyze, and create a wide variety of patterns and functions using appropriate materials and representations in real world problem solving.

**Learning Expectations:**

The student will:

2.1 communicate the meaning of variables in algebraic expressions, equations, and inequalities;
2.2 identify dependent and independent variables in real-world situations;
2.3 apply the concept of variable in simplifying algebraic expressions, solving equations, and solving inequalities;
2.4 represent the solution set linear equations and inequalities in one variable symbolically, graphically, and verbally;
2.5 interpret graphs that depict real-world phenomena;
2.6 model real-world phenomena using graphs;
2.7 represent functions with equations, graphs, tables, and words;
2.8 understand and apply slope as rate of change;
2.9 solve real-world problems represented by linear functions and interpret the slope and intercepts;
2.10 solve systems of two equations in two unknowns using a variety of techniques;
2.11 recognize and extend numerical, geometric, and spatial patterns;
2.12 describe the domain and range of functions imposed either by operations or by real-life situations that the functions represent;
2.13 describe the transformation of the graph that occurs when coefficients and/or constants of the corresponding linear equation are changed;

2.14 generalize numerical, geometric patterns verbally and symbolically.

**Student Performance Indicators:**

**At Level 1, the student is able to**

- extend a geometric pattern;
- extend a numerical pattern;
- translate a verbal expression into an algebraic expression;
- evaluate a first degree algebraic expression given values for one or more variables;
- solve one- and two-step linear equations using integers (with integral coefficients and constants).

**At Level 2, the student is able to**

- select the algebraic notation which generalizes the pattern represented by data in a given table;
- translate a verbal sentence into an algebraic equation;
- select the graph that represents a given linear function expressed in slope-intercept form;
- solve multi-step linear equations (more than two steps, variables on only one side of the equation);
- solve multi-step linear equations (more than two steps, with variables on both sides of the equation);
- solve multi-step linear equations (more than two steps, with one set of parentheses on each side of the equation);
- select the linear graph that models the given real-world situation described in a narrative (no data set given);
- select the linear graph that models the given real-world situation described in a tabular set of data;
- evaluate an algebraic expression given values for one or more variables using grouping symbols and/or exponents less than four;
- determine the slope (rate of change) from the graph of a linear equation (no labeled points);
- apply the concept of rate of change to solve real-world problems;
- select the appropriate graphical representation of a given linear inequality;
- select the non-linear graph that models the given real-world situation or vice versa;
- identify the graphical representation of the solution to a one variable inequality on a number line;
- produce an equation to describe the relationship between data sets;
- explore patterns including Pascal's Triangle and a Fibonacci sequence;
- solve a system of two linear equations using the graphing, elimination, and substitution methods;
- defend the selection of a method for solving a system of equations;
- represent algebraic expressions and operations using manipulatives;
- model the steps for solving simple linear equations using manipulatives;
- write an equation that symbolically expresses a problem solving situation;
- justify correct results of algebraic procedures;
- distinguish between a function and other relationships.

**At Level 3, the student is able to**

- solve multi-step linear inequalities in real-world situations;
- analyze "families of functions" using technology;
- determine the domain and/or range of a function represented by the graph of real-world situations;
- select the system of equations that could be used to solve a given real-world problem;
- find the solution to a quadratic equation given in standard form (integral solutions and a leading coefficient of one).

**Sample Task:**

Students use an almanac or the internet to find the area and the average depth of the world's ten largest bodies of salt water. Then they draw a scatterplot showing the relationship between these two sets of data, and describe the relationship and determine if it is a functional relationship.

**Linkages:**

Mathematics – Probability and Statistics. Patterns in other disciplines and in a variety of cultures.

**Standard 3.0: Geometry**

Students will investigate, model, and apply geometric properties and relationships.

**Learning Expectations:**

The student will:

3.1 apply inductive reasoning in making conjectures, then test conjectures and/or determine a counterexample;

3.2 apply properties of special pairs of angles (e.g. supplementary, complementary, vertical, and adjacent);

3.3 articulate relationships of angles formed when parallel lines are cut by a transversal;

3.4 apply the concept of slope to parallel and perpendicular lines;

3.5 solve real world problems involving length, perimeter, and circumference;

3.6 apply the properties of congruence and similarity to solve problems;

3.7 apply the Pythagorean Theorem and the distance formula;

3.8 use appropriate measurement techniques and tools in investigating properties of polygons (triangle angle properties, angles of polygons, and triangle inequalities).

**Student Performance Indicators:**

**At Level 1, the student is able to**

- describe real-world uses of geometric formulas and relationships;
- discuss issues related to estimating areas of irregular-shaped figures for real-world uses (i.e. fencing, painting, laying carpet, purchasing wallpaper or border);
- identify ordered pairs in the coordinate plane.

**At Level 2, the student is able to**

- apply the given Pythagorean Theorem to a real life problem illustrated by a diagram (no radicals in answer);
- apply proportion and the concepts of similar triangles to find the length of a missing side of a triangle.

**At Level 3, the student is able to**

- calculate the distance between two points given the Pythagorean Theorem and the distance formula;

- determine the height of an object that is difficult to measure by using the properties of similar triangles.

**Sample Task:**

Approximate the value of pi ($\pi$) by looking at the relationship between the diameter and circumference of various circular objects after measuring using a string or a tape measure. Students research and write about how various geometric properties are used in careers such as construction, drafting, and surveying.

**Linkages:**

Mathematics - Estimation, Measurement, and Computation, Research, and the geometric applications in art.

**Standard 4.0: Measurement**

The student will choose appropriate techniques and tools to measure quantities in order to meet specifications for precision and accuracy;

**Learning Expectations:**

The student will

4.1 choose appropriate techniques and tools to measure quantities in order to meet specifications for precision and accuracy;

4.2 use concepts of length, area, and volume to estimate and solve real- world problems;

4.3 apply measurement concepts, relationships, and formulas in algebraic and geometric problem-solving situations;

4.4 use estimation to make predictions and determine reasonableness of results;

4.5 demonstrate an understanding of rates and other derived and indirect measurements (e.g. velocity, miles per hour, revolutions per second, and cost per unit).

**Student Performance Indicators:**

**At Level 1, the student is able to**

- estimate the area of irregular geometric figures on a grid;
- calculate rates involving cost per unit to determine the best buy (no more than three samples)'
- apply the given formula to determine the area or perimeter of a rectangle.

**At Level 2, the student is able to**

- apply the given formula to find the area of a circle, the circumference of a circle, or the volume of a rectangular solid;
- defend estimates of the perimeter and/or area of rectangles and triangles.

**At Level 3, the student is able to**

- select the area representation for a given product of two one-variable binomials with positive constants and coefficients.
- describe how changes in the dimensions of figures affect perimeter, area, and volume.

**Sample Task:**

Place students in small groups giving each group a different length of string. Have each group form a rectangle with the string. Ask each group to measure the sides of their rectangle and find its area. Using the string, direct each group to construct the rectangle with the greatest possible area. Give each group the opportunity to justify their solution.

**Linkages:**

Mathematics – Geometry. Use formulas in Science. Discuss connections to drafting and carpentry. Connect estimation and computation strategies to business and finance.

**Standard 5.0: Data Analysis and Probability**

**Learning Expectations:**

The student will

5.1 collect, represent, and describe linear and nonlinear data sets developed from the real world using appropriate technology;

5.2 choose, construct, and analyze appropriate graphical representations for a data set;

5.3 interpret data using the appropriate measure of central tendency for the data set;

5.4 determine the measures of dispersion of a data set including range and quartiles;

5.5 apply basic counting principles, introducing factorial notation; apply experimental and theoretical probability with simulations where appropriate;

5.6 make predictions from a linear data set using a line of best fit.

**Student Performance Indicators:**

**At Level 1, the student is able to**

- determine the mean (average) of a given set of real-world data (no more than five two-digit numbers);
- interpret bar graphs representing real-world data;
- interpret circle graphs (pie charts) representing real-world data.

**At Level 2, the student is able to**

- graph real-world data using a variety of representations;
- choose the matching linear graph given a set of ordered pairs;
- make a prediction from the graph of a real-world linear data set;
- determine the median for a given set of real-world data (even number of data).

**At Level 3, the student is able to**

- apply counting principles of permutations or combinations in real-world situations;
- debate possible conclusions that can be supported by the data;
- make predictions from real-world data using a line of best fit.

**Sample Task:**

Students research the age of each Tennessee governor at the time of his/her inauguration. The students organize their information and will determine which measure of central tendency is the best description of the data. Students explain their decision.

**Linkages:**

Mathematics - Patterns, Functions, and Algebraic Thinking. Analyze census data. Research and discuss the careers that require the use of statistics such as statistician, actuaries, and scientists.